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NUMERICAL SIMULATION OF HEAT TRANSFER IN MULTILAYER STRUCTURES WITH GENERALIZED NONIDEAL CONTACT

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A procedure is proposed for calculation of a class of problems on heat transfer in multilayer structures with generalized nonideal contact. The procedure is illustrated by calculating the heat transfer in a stack of plates with liquids moving in gaps between the plates.

A typical feature in the design of many technological apparatuses of heat exchangers, power plants, aircraft and rocket technology, etc. is the presence of multilayer components subjected to thermal and mechanical loads. Such components are characterized by different layer thicknesses, different thermophysical characteristics, dependence on temperature in the general case, as well as by thermal resistances at the joints of layers.

Contact thermal resistance depends on many factors: the mean temperature in the zone of contact, the load on the contacting surfaces, the magnitude of the heat flux, the thermophysical properties of the medium occupying the intercontact gaps, the thermophysical and mechanical properties of the contacting surfaces, etc. [1, 2].

In the general case, the zone of contacting layers can contain interlayers of one or another kind of substance (a thin layer of a lubricant, a layer of a moving liquid (heat exchangers, probes) and so on) that can undergo different changes (physicochemical transformations, evaporation or solidification, convective transfer, etc.) with a change in the surface temperature of the layers which are accompanied by heat release or absorption. Under these conditions, there are temperature and heat-flux discontinuities with passage from layer to layer.

Considering the presence of heat-flux discontinuities at the joints of layers, it is reasonable to rewrite the equation relating a temperature discontinuity to the heat flux in a form that is symmetrical relative to the heat fluxes. Then the equations at the joints of layers can be written in general form as the following equalities

$$T_j^+ - T_j^- = R_j^+ \left(\lambda \frac{\partial T}{\partial r}\right)_j^+ + R_j^- \left(\lambda \frac{\partial T}{\partial r}\right)_j^- ,$$
$$\left(\lambda \frac{\partial T}{\partial r}\right)_j^+ - \left(\lambda \frac{\partial T}{\partial r}\right)_j^- = \omega_j , \quad r = r_{j-1} , \quad j = \overline{1, k-1} . \tag{1}$$

Here R_j^+, R_j^- are the coefficients of the generalized contact thermal resistance at the boundary of the $j+1$-th and j-th layers.

In the special case of $R_j^+ = R_j^- = 1/2R_j$ and $\omega_j = 0$, the problem on heat transfer in a multilayer structure with nonideal thermal contacts has the usual formulation [2]. Generalized conditions (1) make it possible within the framework of a single algorithm to solve a wide class of problems on heat transfer complicated by various accompanying processes between the surfaces of contacting layers. In this case, both direct thermal and mechanical contacts of these surfaces and their thermal contact via a heat-transfer agent can take place. In the latter case, direct mechanical contact of these surfaces can be absent.

To formulate a unified algorithm and develop software for numerical solution of a wide class of problems, the equations describing unsteady heat transfer at internal points of layers and the boundary conditions over the outer surfaces of a structure can be written in the following unified form

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$$\frac{\partial (b_j T)}{\partial \tau} = c_j \left[\left(\frac{v d_j}{r} \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial r} \left(d_j \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(d_j \frac{\partial T}{\partial z} \right) \right] + e_j , \quad j = \overline{1, k} , \tag{2}$$

$$b^{(\bullet)}_{1.4} + b^{(\bullet)}_{2.5} \frac{\partial T}{\partial r} = b^{(\bullet)}_{3.6} \tag{3}$$

at $r = r_1$ and $r = r_{k+1}$ $z_0 < z < z_1$; $z = z_0$ and $z = z_1$ $r_1 < r < r_{k+1}$.

By formulating algorithms for each particular case or expressions for determination of the coefficients of differential equations, parameters of generalized nonideal contact, and boundary conditions, one can consider a large group of problems on heat transfer in multilayer structures within the framework of a single methodological approach. In particular, using various methods to determine the coefficients of boundary conditions, we can consider heat transfer in external flow around a surface, in gas and liquid flows in tubes and channels, and in the presence of phase changes, physicochemical transformations, etc.

In what follows the problem described by system of differential equations (2), conditions (1) at the boundaries of contacting layers, and boundary conditions (3) will be spoken of as the problem on heat transfer in a multilayer structure with generalized nonideal thermal contact or with generalized thermal resistance.

To solve numerically the problem on heat transfer in a multilayer structure with generalized nonideal contact, we can formulate a homogeneous algorithm. For this, use is made of finite-difference schemes of implicit approximation with absolute time stability.

Using one or another scheme for finite-difference approximation of Eqs. (2) to determine the temperature values at the inner nodes of the difference network that do not lie at the boundaries of layers, we arrive at a system of algebraic equations with a three-diagonal matrix [3]

$$A_i T_{i+1} + B_i T_i + C_i T_{i-1} = D_i . \tag{4}$$

The coefficients A, B, C, D of these equations are expressed in terms of the coefficients of initial differential equations (2).

To close system of Eqs. (4), it is necessary to supplement it with equations that are a difference approximation of boundary conditions (3) and the relations at the joints of layers (1).

On the surface separating these layers, two nodes i^+ and i^-, located, respectively, in the $(j+1)$-th and j-th layers, are introduced instead of the i-th node on this surface. In the case of generalized nonideal thermal contact, $r_i^+ = r_i^- + \delta_j$, where δ_j is the gap between the adjacent surfaces of the j-th and $(j+1)$-th layers.

For approximation of Eq. (1) at the i_j-th node on the boundary surface of layers, fictitious nodes are introduced which lie on the extended j-th and $(j+1)$-th layers, respectively. After elimination of the temperature values at the fictitious nodes, from the difference approximation of Eqs. (1) with the aid of (4) we obtain the following two equations:

$$\begin{gathered} A_i^- T_{i+1} + B_i^- T_i^- + C_i^- T_{i-1} = D_i^- , \quad i = i_j , \quad j = \overline{1, k-1} ; \\ A_i^+ T_{i+1} + B_i^+ T_i^+ + C_i^+ T_{i-1} = D_i^+ , \quad i = i_j , \quad j = \overline{1, k-1} . \end{gathered} \tag{5}$$

The coefficients of these equations are expressed in terms of the parameters of generalized nonideal contact (1) and the coefficients of Eqs. (4) for $i = i_j - 1$ and $i = i_j + 1$.

Similarly, using fictitious nodes and relations (4) at $i = 1$ and $i = k$ for the finite-difference approximation of the boundary conditions, an absolutely stable implicit scheme can be proposed:

$$T_{1,N} = L_{1,N} T_{2,N-1} + K_{1,N} , \tag{6}$$

where the coefficients $L_{1,N}$ and $k_{1,N}$ are expressed in terms of the coefficients of Eqs. (4) for $i = 1$, N and the coefficients of boundary conditions (3).

As analytical and numerical studies show, this approximation, unli'-e those usually employed [3, 4], is absolutely stable and allows results to be obtained under conditions of intense external heat supply.

Thus, for temperature determination we have system of difference equations (4), (5), and (6), which approximates the initial mathematical problem with an error of order $O(\Delta r^2, \Delta \tau)$.

In the general case, the parameters ρ, c, λ and, consequently, the coefficients A, B, C, D depend on T, i.e., this system is linear and for its solution an iteration process is used, at each cycle of which the coefficients of the equations are calculated for the temperature values taken from the previous iteration, and the linear system of equations is solved.

It is easy to see that in the case of a single-layer structure or a multilayer structure with ideal thermal contact between layers, $A_i^+ = A_i^-$, ..., $D_i^+ = D_i^-$. Then, with allowance for $T_i^+ = T_i^-$, Eqs. (5) degenerate into one equation and system of Eqs. (4)-(6) becomes a system of algebraic equations with a three-diagonal matrix that is effectively solved by the elimination method [5, 6]. Problems of finite-difference approximation of the conditions at the layers boundaries in these cases are discussed in [3].

However, in the general case, for the problem with generalized nonideal contact $T_i^+ \not\equiv T_i^-$ and $A_i^+ \not\equiv A_i^-$, ..., $D_i^+ \not\equiv D_i^-$, and it is necessary to solve the complete system of Eqs. (4)-(6), whose matrix differs from the three-diagonal one. This prevents direct use of the elimination method.

In order to employ this method, we reformulate this system of equations into two systems of equations: a system of equations for T_j, T_j^+ and relations expressing T_i^- in terms of T_i^+ and T_{i-1}.

The second system, consisting of $k - 1$ equations, can be derived from (5) after elimination of T_{i+1} from it. After trivial transformations we arrive at

$$T_i^- = K_i T_i^+ + L_i T_{i-1} + N_i . \tag{7}$$

Eliminating T_i^- from Eq. (4) with the aid of (7) at $i = i_j - 1$ and combining like terms, we obtain

$$\overline{A}_{i_j-1} T_{i_j-2} + \overline{B}_{i_j-1} T_{i_j-1} + \overline{C}_{i_j-1} T_{i_j}^+ = \overline{D}_{i_j-1} . \tag{8}$$

Thus, we have system of Eqs. (4), (6), the second equation of (5), and Eq. (8) written in the unknown T_i (the temperature at the inner nodes of the difference network) and T_j^+ (the temperature at a point on the lower surface in the $j + 1$-th layer).

In each iteration cycle this system of equations is a system of linear algebraic equations with a three-diagonal matrix that can be solved by the three-point elimination method. After T_i, T_i^+ determination, T_i^- on the upper surface of the j-th layer is determined from Eq. (7).

To illustrate the adopted approach, we will consider the problem of heat transfer calculation in a stack of plates with a liquid flowing between the latter. Under such conditions the following conditions will be fulfilled on the contacting surfaces of adjacent plates:

$$\begin{gathered} - \left(\lambda \frac{\partial T}{\partial r}\right)_j^- = \alpha_j^- \left(T_j^- - T_{f_j}\right), \quad r = r_{j+1}^- ; \\ - \left(\lambda \frac{\partial T}{\partial r}\right)_j^+ = \alpha_j^+ \left(T_j^+ - T_{f_j}\right), \quad r = r_{j+1}^+ , \quad j = \overline{1, k} . \end{gathered} \tag{9}$$

After trivial transformations that reduce (9) to a form corresponding to generalized nonideal contact (1), we arrive at relations allowing determination of the generalized nonideal contact parameters R_j^+, R_j^+, ω_j:

$$R_j^+ = \frac{-1}{\alpha_j^+}, \quad R_j^- = \frac{-1}{\alpha_j^-}, \quad \omega_j = \alpha_j^- \left(T_j^- - T_{f_j}\right) + \alpha_j^+ \left(T_{f_j} - T_j^+\right), \tag{10}$$

which are independent of the method used for determination of heat transfer coefficients.

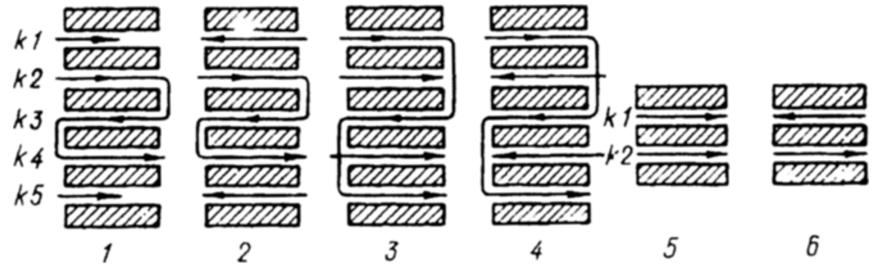


Fig. 1. Schemes of liquid flows: 1-6, Nos. of schemes; $k1$ – $k5$, Nos. of channels.

Thus, relations (10) determine the generalized thermal resistance parameters when the adjacent surfaces of the elements of a multilayer structure are separated by a moving heat-transfer agent.

To calculate the temperature of the heat-transfer agent, we use a quasi-one-dimensional model [1]:

$$\rho_f c_{p_f}\left(\frac{\partial T_{f_j}}{\partial \tau} + u\frac{\partial T_{f_j}}{\partial z}\right) = \frac{1}{\delta_j}\left[\alpha_j^+ (T_j^+ - T_{f_j}) + \alpha_j^- (T_j^- - T_{f_j})\right], \tag{11}$$
$$j = \overline{1, k-1}, \quad z_0 < z < z_1 .$$

To solve this equation numerically, the absolutely stable running-count difference scheme is applied using finite differences oriented against the flow, which requires consideration of the flow direction in each channel:

$$\rho_{f_i} c_{p_{f_i}}\left(\frac{T_{f_i}^{p+1} - T_{f_i}}{\Delta\tau} + |u|\frac{T_{f_i} - T_{f_{i-\mathrm{sign}|u|\cdot 1}}}{\Delta z}\right) = \frac{1}{\delta_j}\left(-(\alpha_i^+ + \alpha_i^-)T_{f_i} + \alpha_i^+ T_j^+ + \alpha_i^- T_j^-\right). \tag{12}$$

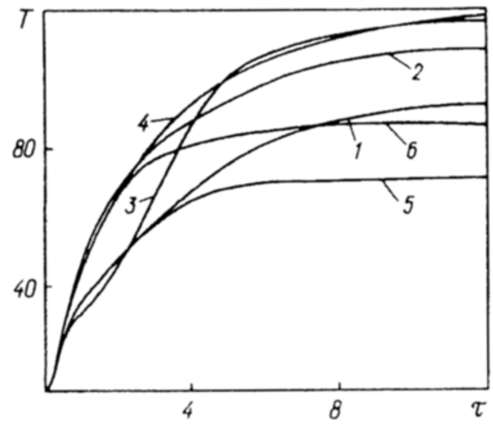
Data in the inlet sections of the channels are either given or determined when solving the problem with allowance for the specific features of liquid inflow into the channel considered, depending on the specific diagram of the structure.

To construct a formalized algorithm that is independent of the number of channels and the flow diagrams of the heat-transfer agents, a matrix is introduced to formalize the conditions at joints, each row of which contains information about the parameters of the heat-transfer agent flow in the corresponding channel.

In the first element of a row, a parameter *js* is assigned to indicate the number of the layer above which the fluid flows. Introduction of this parameter allows a calculation to be made within the framework of a single algorithm when the structural elements between two layers of fluid represent a layer of the same material and also when an element itself consists of several layers, the thermal resistance of which is different. If all the structural elements between two layers fluid are homogeneous, then *js* is equal to the number of the current matrix row. In the second element, the number of the channel, i.e., the parameter *jf*, out of which the fluid flows is prescribed. In the third, the parameter *kf* is specified, i.e., the number of structure channels through which the fluid has passed prior to entering the channel under consideration. In the fourth, u_{in}, the specific mass flow rate of the fluid. In the fifth, $T_{f,\mathrm{in}}$, the fluid temperature in the inlet section. In specifying the flow rate, a sign is used to indicate for the flow direction. The quantities u_{in} and $T_{i,\mathrm{in}}$ are prescribed concretely for the fluid with $kf = 0$. At $kf \neq 0$ these parameters are determined when solving the problem numerically.

To illustrate the possibilities of the formulated algorithms, we solved the problem on heating (cooling) of a liquid in a structure consisting of six layers. Flow diagrams of the liquids are given in Fig. 1. For compound schemes (1-4), the temperature of the fluids flowing into two separate channels, $T_{f,\mathrm{in}}$ is 150°C, the inlet temperature of the fluid flowing, as in a coil, in three other communicating channels is 10°C. In addition to these schemes, we also considered, for comparison sake, simple schemes containing two channels (5, 6).

The results reported below were obtained for the following parameters: the channel length $L = 2.5$ m, the wall thickness of the structure $h_j = 3$ mm, and the width of the channels in which fluid flows is $\delta_j = 1.5$ mm. The



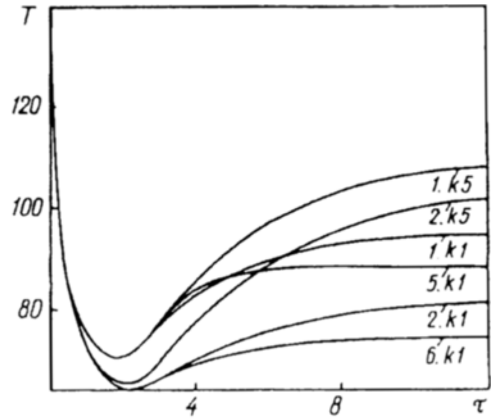


Fig. 2. Temperature variation of heated fluid at structure outlet. The number of the lines indicates the number of the scheme. T, °C; τ, sec.

Fig. 3. Temperature variation of the heating fluids at structure outlet. The number of the line indicates the number of the scheme, the channel.

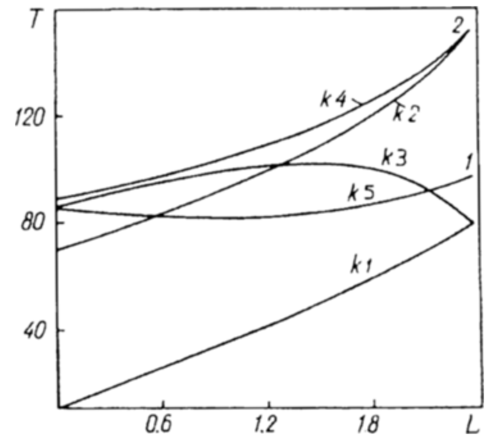


Fig. 4. Temperature variation of fluids along structure channels. The fluids flow according to scheme 4, τ = 6 sec. Line 1 corresponds to the heated fluid, line 2 – to the heating fluid. The figures above the lines indicate the numbers of the channels. L, m.

channel walls were made of steel, λ_j = 46.5 W/m^2/deg, $a_j = 5.9 \cdot 10^{-6}$ m^2/sec. The initial temperature of the structure was 10°C. Water was used as the fluid. Calculations were made with allowance for the temperature dependence of the density and thermophysical characteristics of water. These parameters were determined by interpolation of the table values [7]. The mass flow rate of water is 1200 kg/sec/m^2. It is assumed that the pressure in the system of channels is higher than that of saturated steam at T_f = 150°C, i.e., phase changes are neglected.

The time variation of the heating and heated liquids at the structure outlet is shown in Figs. 2 and 3. As is seen, as regards the heating of liquids, a scheme in which cold liquids are flowing counter to a hot liquid (schemes 2 and 4) is more effective, as in the simple case of liquids flowing in two channels, than a scheme with a cocurrent flow (schemes 1 and 3). Comparing schemes 1 and 3 and 2 and 4, it is pertinent to note that at the initial moment (τ < 2 sec) the trend of temperature variation of the heated liquid is practically the same, since at this period the influence of the heating liquids in channels $k1$ (scheme 1, 2) and $k2$ (scheme 3, 4) is insignificant. The temperature of the outflowing liquid is mainly affected by heat transfer between channels $k4$ and $k5$ independently of their lay-out diagram. The schemes with counter and cocurrent flows differ considerably. In this period, heating of a cold liquid in the outlet channel proceeds for compound schemes in the same way as in a simple system (following a scheme of counter or cocurrent flow). However at the next moments of time (2 ≤ τ ≤ 4 sec) heating of the liquids in channels $k1$ and $k2$ plays a more significant role. This, in particular, indicates an abrupt increase in th· slope of the curve that illustrates the temperature variation of a liquid flowing according to scheme 3.

The time dependence of the temperature of hot liquids at the structure outlet is shown in Fig. 3 for schemes 1 and 2. Noteworthy is the nonmonotonic character of its variation. At the initial moment ($\tau \le 2$ sec) cooling of the hot liquid by the cold one prevails. It should be noted that liquid cooling in the system under consideration proceeeds in the same way as in a simple system (the corresponding curves for these systems practically coincide). Then a rise in the cold-liquid temperature over the entire length of the channels (Fig. 4) causes considerably less cooling of the hot liquid.

The temperature variation of the liquids along the channels at $\tau = 6$ sec for scheme 4 is shown in Fig. 4.

In conclusion, it should be noted that though the scheme of the considered structures is rather complicated, the algorithm is possibilities are not exhausted by the examples discussed. These algorithms are of a sufficiently general nature and can be used for calculation of the thermal states of structures that are of practical interest.

NOTATION

τ, time; T, temperature; λ, thermal conductivity of the materials of structure layers; e, specific power of the heat sources (sinks) of the materials of the structure layers; r and z, coordinates in the transverse and longitudinal directions; ν, parameter designating the coordinate system, R^+, R^-, ω, coefficients of generalized nonideal contact; T_f, mass-averaged fluid temperature; c_{pf}, ρ_f, specific heat per unit mass and fluid density at temperature T_f, respectively; a, heat transfer coefficient; u, mean flow rate; G, mass specific flow rate of the fluid; δ, channel width; L, structure length; p, number of time layer; k, number of structure layers. The subscript j indicates the layer number, the superscripts "+" or "−" refer the parameters "from above" and "from below" of joints, respectively.

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